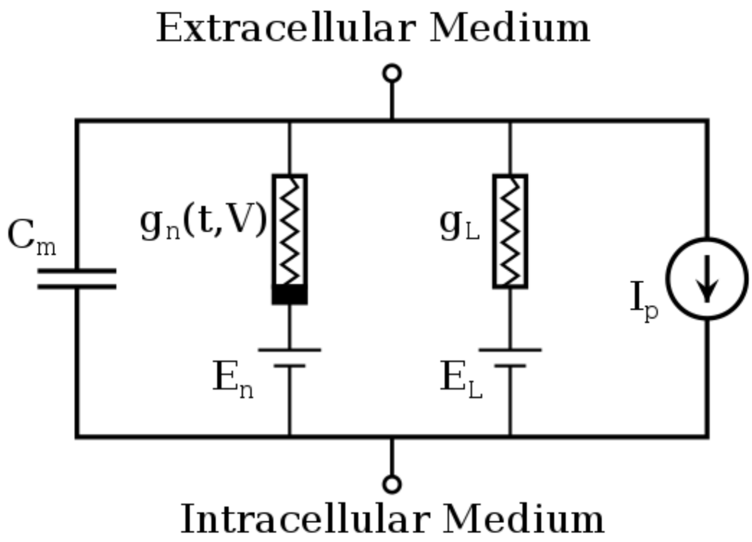
Outline

- HH Model Revisited
- Nonlinear Dynamics in HH Model

HH Model Revisited

Introduction



(figure from Internet)

In HH model, we use 3 additional ODEs to replace the 2 imposed eqs of IF model. And its first ODE is also gotten from its circuit diagram.

$$I = C_m \frac{\mathrm{d}V_m}{\mathrm{d}t} + \bar{g}_\mathrm{K} n^4 (V_m - V_K) + \bar{g}_\mathrm{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l)$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1-n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1-m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1-h) - \beta_h(V_m)h$$

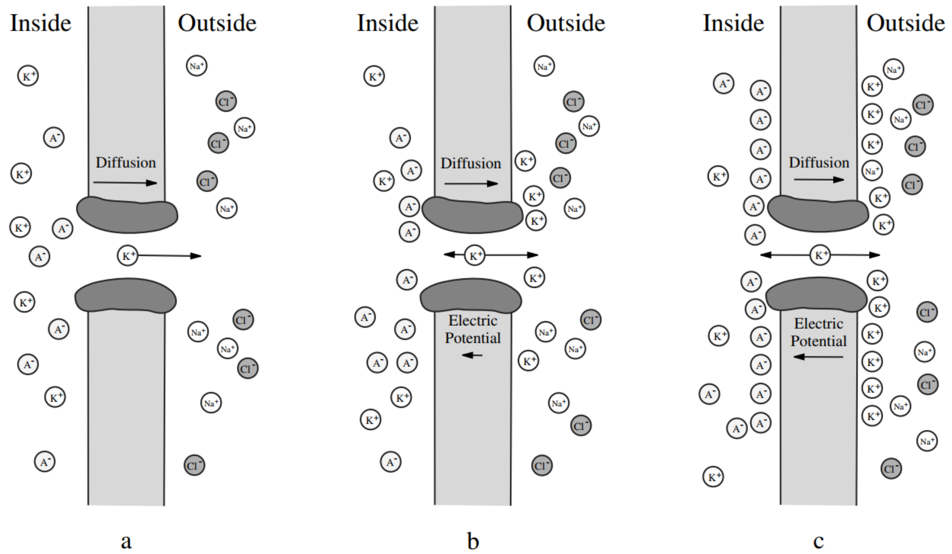
(figure from Internet)

Physical Perspective

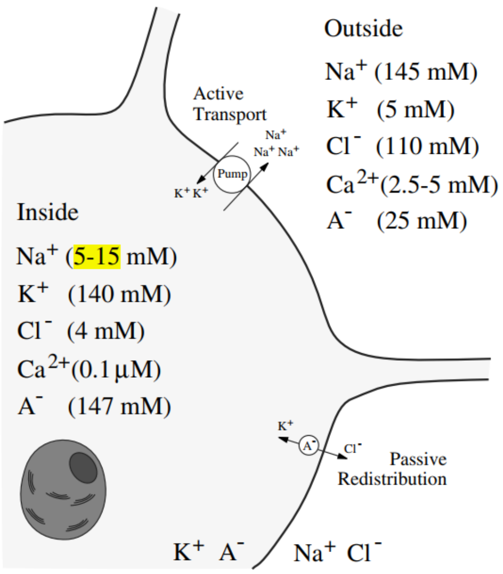
- What is the meaning of α and β ?
  - Transition probability in the 2 state Markov chain.
- Why the exponent is $4, 3, 1$ for n, m, h in the first ODE?
  - They come from fitting the experiment data.

Biological Perspective

There are a few things you need to know:

- How to calculate the V_K and V_{Na}? (Sometimes they are called Nernst Equilibrium Potential)
    - Method 1: using diffusion equation
        - 

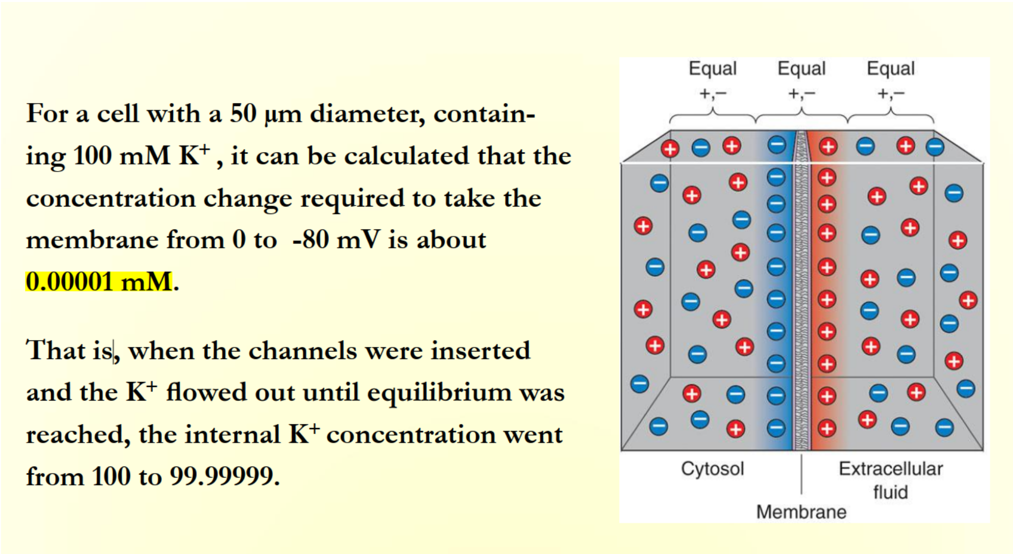
    - Method 2: using Boltzmann distribution
- What is the quantity of Nernst Equilibrium Potential of typical ions?



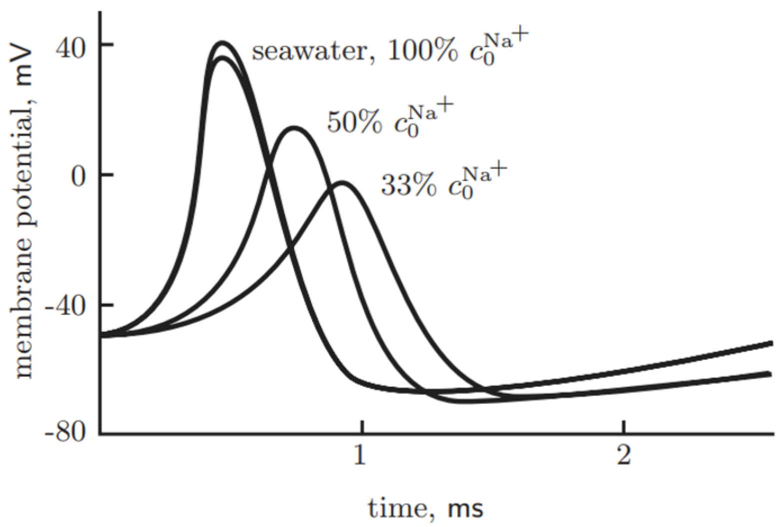
Equilibrium Potentials

$$
\begin{aligned}
Na^+ \quad & 62\log\frac{145}{5} = 90\text{ mV} \\
& 62\log\frac{145}{15} = 61\text{ mV} \\
K^+ \quad & 62\log\frac{5}{140} = -90\text{ mV} \\
Cl^- \quad & -62\log\frac{110}{4} = -89\text{ mV} \\
Ca^{2+} \quad & 31\log\frac{2.5}{10^{-4}} = 136\text{ mV} \\
& 31\log\frac{5}{10^{-4}} = 146\text{ mV}
\end{aligned}
$$

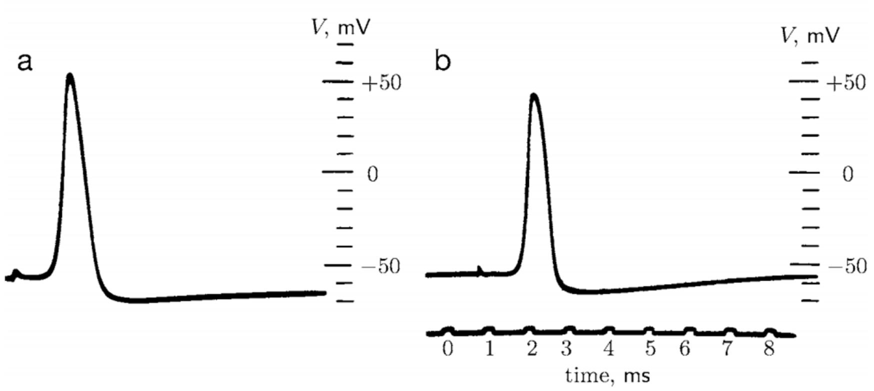
Ion	Concentration outside (in mM)	Concentration inside (in mM)	Ratio Out : In	E_{ion} (at 37∞C)
K^+	5	100	1 : 20	–80 mV
Na^+	150	15	10 : 1	62 mV
Ca^{2+}	2	0.0002	10,000 : 1	123 mV
Cl^-	150	13	11.5 : 1	–65 mV

- What are the concentration of Na^+, K^+, Cl^-, Ca^{2+} inside the neuron and outside the neuron?
    - see above.
- How much will they change during the action potential?
    - K^+
        - 

    - Ca^{2+}: ???
- What does the $Na^+ - K^+$ *pump* do in the action potential? Is it the same thing as the ion channel?
    - hyper-polarization.
    - no.
- 3 ways for Na^+, K^+ to pass the membrane
    - leaky
    - ion channel
    - ion exchanger/ion pump
- Why HH only used Na^+ and K^+?



(Figure 12.7 of *Biological Physics*)



left panel: an axon filled with potassium sulfate solution.

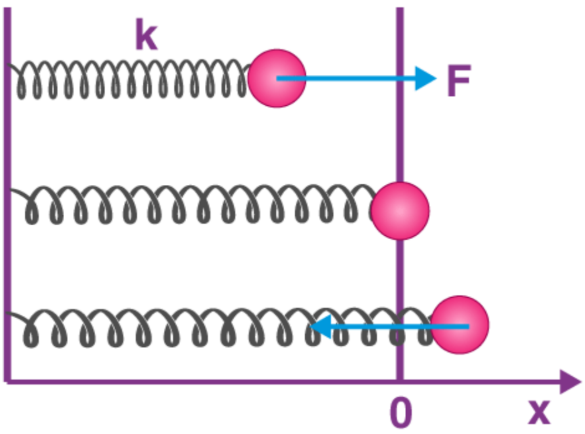
right panel: an normal axon.

(Figure 12.13 of *Biological Physics*)

Nonlinear Dynamics in HH Model

Introduction

Eg1: harmonic oscillator



Eg2: simple pendulum

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \pm\sqrt{2a^2 \cos x - C_1}.$$

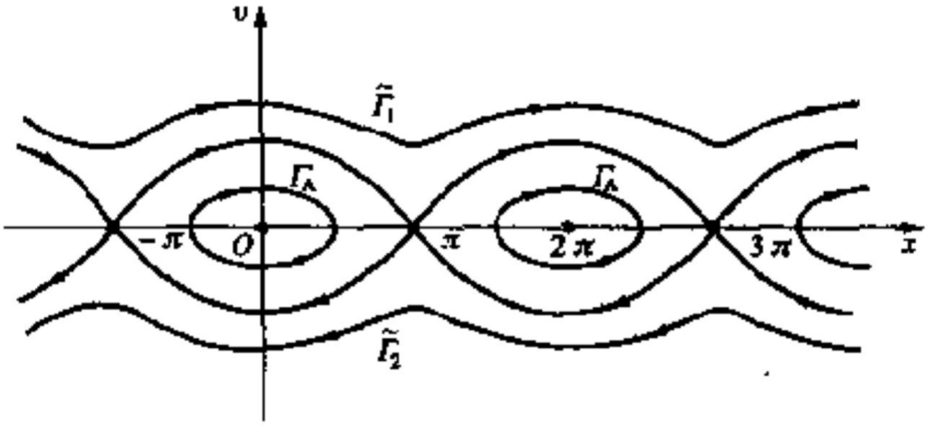


图 5-5

Fixed Point

- nullcline
- vector field

Limit Cycle

- Poincare-Bendixson TH